

# CP, T VIOLATION IN NEUTRINO OSCILLATIONS

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## **Abstract**

The manifestation of CP, T violation in the leptonic sector is studied for flavour neutrino oscillations, both in vacuum and in matter. Different conditions of short-base-line versus long-base-line experiments are discussed.

# 1 Introduction

Complex neutrino mixing for 3 family Dirac neutrinos leads to CP and T violation effects in neutrino oscillations [1]. In view of the vigorous experimental programme in this field, the study of CP violation becomes an interesting topic.

The neutrino states of definite flavour  $\alpha$ , as generated by well defined weak interaction properties, are related to neutrino states of definite mass  $m_k$  by

$$\nu_\alpha = \sum_k U_{\alpha k} \nu_k \quad (1)$$

where  $U$  is the unitary mixing matrix which, for 3 families, depends on 3 mixing angles and 1 CP phase.

If the " $\alpha$ " state is prepared at  $t = 0$ , the probability amplitude that, at time  $t$ , it is manifested as the " $\beta$ " state is

$$A(\alpha \rightarrow \beta; t) = \sum_k U_{\alpha k} U_{\beta k}^* \exp[-iE_k t] \quad (2)$$

We observe that the time-dependent amplitude contains the interference of different " $k$ " terms, with different weak phases in  $U_{\alpha k} U_{\beta k}^*$  and different oscillation phases governed by  $E_k$ . These ingredients are necessary and sufficient to generate CP violation in the oscillation probability.

In Section 2 we discuss the CPT-invariance condition, together with the CP-odd and T-odd asymmetries. In Section 3 the CP-asymmetry for 3 family neutrino oscillation is considered and the conditions for a non-vanishing value are obtained. These results will lead to the need of long-base-line (LBL) experiments for CP studies. In Section 4 the CP-odd asymmetry is built in this case for hierarchical neutrino masses. These LBL experiments have to include, however, matter effects and in Section 5 we show that these matter effects are large and they constitute an undesired background for CP violation effects. Due to this fake phenomenon, Section 6 studies T-odd asymmetries which are free from this problem. Section 7 answers the question related to the possible Majorana character of neutrinos. Section 8 summarizes some conclusions and the outlook.

## 2 CPT, CP, T

From Eq. (2) the requirement of CPT invariance leads to the amplitude for conjugate flavour states

$$A(\bar{\alpha} \rightarrow \bar{\beta}; t) = \sum_k U_{\alpha k}^* U_{\beta k} \exp[-iE_k t] \quad (3)$$

so that we obtain the condition

$$CPT \Rightarrow A(\bar{\alpha} \rightarrow \bar{\beta}; t) = A^*(\alpha \rightarrow \beta; -t) \quad (4)$$

Eq. (4) will be assumed through this work.

CP-invariance is the statement that the probabilities for the original transition and for its conjugate are equal, i.e.,

$$CP \Rightarrow |A(\alpha \rightarrow \beta; t)|^2 = |A(\bar{\alpha} \rightarrow \bar{\beta}; t)|^2 \quad (5)$$

T-invariance is the statement that the probabilities for the original transition and for its inverse are equal, i.e.,

$$\begin{aligned} T \Rightarrow |A(\alpha \rightarrow \beta; t)|^2 &= |A(\beta \rightarrow \alpha; t)|^2 \\ |A(\bar{\alpha} \rightarrow \bar{\beta}; t)|^2 &= |A(\bar{\beta} \rightarrow \bar{\alpha}; t)|^2 \end{aligned} \quad (6)$$

From these results, we have the corollaries:

i) CP, T Violation effects, i.e, the violation of Eq. (5) , Eq. (6), respectively, can take place in Appearance Experiments only. For Disappearance experiments,  $\beta = \alpha$ , Eq. (6) is automatic and Eq. (2) implies

$$A^*(\alpha \rightarrow \alpha; t) = A(\alpha \rightarrow \alpha; -t) \quad (7)$$

The combination of Eq. (7) and Eq. (4) leads to the verification of Eq. (5), q.e.d. As a consequence, no CP or T violation effect can be manifested in reactor or solar neutrino experiments.

ii) The (numerator of) CP-odd Asymmetry is given by

$$D_{\alpha\beta} \equiv |A(\alpha \rightarrow \beta; t)|^2 - |A(\bar{\alpha} \rightarrow \bar{\beta}; t)|^2 \quad (8)$$

CPT-invariance implies  $D_{\alpha\beta} = -D_{\beta\alpha}$  and the use of Eq. (2) and the Unitarity of the Mixing Matrix implies  $\sum_{\beta \neq \alpha} D_{\alpha\beta} = 0$ . These constraints lead to a unique  $D$  for 3 flavours:

$$D_{e\mu} = D_{\mu\tau} = D_{\tau e} \quad (9)$$

iii) The (numerators of) T-odd Asymmetries are given by

$$\begin{aligned} T_{\alpha\beta} &\equiv |A(\alpha \rightarrow \beta; t)|^2 - |A(\beta \rightarrow \alpha; t)|^2 = |A(\alpha \rightarrow \beta; t)|^2 - |A(\alpha \rightarrow \beta; -t)|^2 \\ \bar{T}_{\alpha\beta} &\equiv |A(\bar{\alpha} \rightarrow \bar{\beta}; t)|^2 - |A(\bar{\beta} \rightarrow \bar{\alpha}; t)|^2 = |A(\bar{\alpha} \rightarrow \bar{\beta}; t)|^2 - |A(\bar{\alpha} \rightarrow \bar{\beta}; -t)|^2 \end{aligned} \quad (10)$$

where use of Eq. (2) has been made in the right-hand side. Eq. (10) leads to the important conclusion that  $T_{\alpha\beta}, \bar{T}_{\alpha\beta}$  are odd functions of time. One should be aware that Eq. (10) needs an hermitian Hamiltonian for the evolution of the system. In fact, the above conclusion is not valid for the  $K^0 \bar{K}^0$  system.

### 3 CP-Asymmetry

The numerator Eq. (8) of the CP-odd asymmetries can be calculated using Eq. (2) for the amplitudes. In the limit of ultrarelativistic neutrinos, one obtains

$$D_{\alpha\beta} = \sum_{k>j} I_{\alpha\beta;jk} \sin \frac{\Delta m_{kj}^2 L}{2E} \quad (11)$$

where  $L \simeq t$  is the distance between the source and the detector,  $E$  is the neutrino energy and  $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$ . The  $I$ 's containing mixing angles and the CP phase are given by

$$I_{\alpha\beta;jk} = 4Im[U_{\alpha j}U_{\beta j}^*U_{\alpha k}^*U_{\beta k}] \quad (12)$$

which show the rephasing invariance of the observables explicitly.

Suppose that only the highest  $m^2$ -value is relevant for the neutrino oscillation experiment, assuming a hierarchy in neutrino masses. This statement, which can be considered as the definition of a short-base-line (SBL) experiment, means that the approximations

$$\left. \begin{aligned} \Delta m^2 &\simeq \Delta m_{31}^2 \simeq \Delta m_{32}^2 \\ \frac{\Delta m_{21}^2}{2E} L &\ll 1 \end{aligned} \right\} \quad (13)$$

are fulfilled. In the limit of neglecting terms of order  $\frac{\Delta m_{21}^2}{2E} L$ , the asymmetry Eq. (11) becomes

$$D_{\alpha\beta}^{(SBL)} \simeq (I_{\alpha\beta;13} + I_{\alpha\beta;23}) \sin \frac{\Delta m^2 L}{2E} \quad (14)$$

which vanishes due to the cyclic character of the  $I$ 's:  $I_{\alpha\beta;23} = I_{\alpha\beta;31} = -I_{\alpha\beta;13}$ .

The lesson learnt from this limit is immediate: the 3 families have to participate ACTIVELY in order to generate a non-vanishing CP-odd observable. It is not enough to know that there are 3 non-degenerate neutrinos in Nature and the presence of mixing among all of them.  $\Delta m_{21}^2$  has to participate. Furthermore, in order to generate a non-vanishing value for the  $I$ 's one needs ALL the mixing angles and the unique CP phase different from zero [2].

One thus concludes that a significant CP-odd asymmetry needs the consideration of neutrino oscillations in long-base-line (LBL) experiments [3, 4, 5, 6]. The meaning of this requirement is that both  $\Delta m^2$ 's, i.e.,  $\Delta m_{31}^2 \simeq \Delta m_{32}^2$  and  $\Delta m_{21}^2$ , have to be accessible.

Two comments should be considered to soften the above conclusion: i) one can choose to keep terms of order  $\frac{\Delta m_{21}^2}{2E} L \ll 1$  at the expense to search for appearance transitions with very low probability and enhance the CP-odd ratio which defines the asymmetry; ii) even if the value of  $D_{\alpha\beta}$ , and thus of the CP-odd asymmetry, vanishes under the conditions leading to Eq. (14), the existence of a non-vanishing CP-phase can be inferred from CP-conserving observables if enough probabilities are measured. To have information on both  $P(\nu_\mu \rightarrow \nu_\tau)$  and  $P(\nu_e \rightarrow \nu_\tau)$  under controllable conditions, one probably needs the neutrino facility based on muon-storage-rings [7].

## 4 CP effects in LBL experiments

Contrary to the conditions discussed before in Eq. (13), we assume in this Section that  $L/E$  is such that

$$\left. \begin{aligned} \frac{\Delta m_{31}^2 L}{2E} &\sim \frac{\Delta m_{32}^2 L}{2E} \gg 1 \\ \frac{\Delta m_{21}^2 L}{2E} &\sim 1 \end{aligned} \right\} \quad (15)$$

The calculation of the appearance probabilities  $\alpha \rightarrow \beta$  then gives

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta} &= |U_{\beta 1}^* U_{\alpha 1} + U_{\beta 2}^* U_{\alpha 2} \exp(-i \frac{\Delta m_{21}^2 L}{2E})|^2 + |U_{\beta 3}|^2 |U_{\alpha 3}|^2 \\ P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} &= |U_{\beta 1} U_{\alpha 1}^* + U_{\beta 2} U_{\alpha 2}^* \exp(-i \frac{\Delta m_{21}^2 L}{2E})|^2 + |U_{\beta 3}|^2 |U_{\alpha 3}|^2 \end{aligned} \quad (16)$$

One notices that the heaviest neutrino contributes to these probabilities only through mixing without any oscillation: this term is CP-even. What is relevant for the CP asymmetry is the interference of two amplitudes  $k = 1, 2$  with different weak phases and different oscillation phases: in going to the CP transformed transition, the weak phase changes its sign whereas the oscillation phase remains the same.

The difference of the two probabilities Eq. (16) gives a CP-odd asymmetry

$$\left. \begin{aligned} D_{\alpha\beta}^{(LBL)} &\simeq I_{\alpha\beta} \sin \frac{\Delta m_{21}^2 L}{2E} \\ I_{\alpha\beta} &\equiv I_{\alpha\beta;12} = 4 \text{Im}[U_{\alpha 1} U_{\beta 1}^* U_{\alpha 2}^* U_{\beta 2}] \end{aligned} \right\} \quad (17)$$

It is immediate to realize that, for the 3-neutrino case, one has

$$I_{e\mu} = I_{\mu\tau} = I_{\tau e} \quad (18)$$

Bilenky et al. [6] have used present exclusion plots for  $\mu \rightarrow e$  and  $\mu \rightarrow \tau$  transitions, together with amplitude and unitarity bounds, to find allowed values for  $|I_{e\mu}|$  and  $|I_{\mu\tau}|$ . Maximum values of  $10^{-2}$  for  $|I_{e\mu}|$  and around  $10^{-1}$  for  $|I_{\mu\tau}|$  are accessible.

## 5 Matter effects in LBL experiments

LBL experiments, with source and detector at the earth surface, imply that neutrinos cross the earth in their travel. It is mandatory to discuss the matter effect [8, 9, 10]

The effective Hamiltonians for neutrinos and antineutrinos are given in the flavour basis for 3 families:

$$\begin{aligned} H_\nu &= \frac{1}{2E} \left\{ U \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\} \\ H_{\bar{\nu}} &= \frac{1}{2E} \left\{ U^* \begin{pmatrix} m_1^2 & & \\ & m_2^2 & \\ & & m_3^2 \end{pmatrix} U^T - \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\} \end{aligned} \quad (19)$$

where the matter effect for constant density is given by the forward charged current interaction amplitude with electrons

$$a = G\sqrt{2}N_e 2E \simeq 2.3 \times 10^{-4} eV^2 \left(\frac{\rho}{3gcm^{-3}}\right) \left(\frac{E}{GeV}\right) \quad (20)$$

and  $N_e$  is the (number) density of electrons. It is interesting to build the dimensionless quantity

$$\frac{aL}{2E} \simeq 0.58 \times 10^{-3} \left(\frac{L}{km}\right) \quad (21)$$

which is E-independent! This is in contrast to the E-dependent oscillation quantity which, for  $\Delta m^2$  in the range of the atmospheric neutrino oscillation solution, gives

$$\frac{\Delta m^2 L}{2E} \simeq 0.75 \times 10^{-2} \left(\frac{L}{km}\right) \left(\frac{GeV}{E}\right) \quad (22)$$

For neutrino beams with energy  $E \sim 10 GeV$ , as envisaged by the FermiLab and CERN LBL experiments, the two quantities Eq. (21) and Eq. (22) are comparable and one concludes that large matter effects are expected.

The diagonalization of  $H_\nu$  and  $H_{\bar{\nu}}$  by unitary matrices  $U'$  and  $\bar{U}'$  leads to different eigenvalues  $\tilde{m}_\nu^2$  and  $\tilde{m}_{\bar{\nu}}^2$ , respectively,

$$H_\nu = U' \frac{\tilde{m}_\nu^2}{2E} U'^+ \quad , \quad H_{\bar{\nu}} = \bar{U}'^* \frac{\tilde{m}_{\bar{\nu}}^2}{2E} \bar{U}'^T \quad (23)$$

One notes that the matter effect  $a \neq 0$  provokes fake CP and CPT violation associated with the interaction with the asymmetric medium: even the simplest diagonal probability equality for  $\nu_e$  and  $\bar{\nu}_e$  is violated,  $P_{\nu_e \rightarrow \nu_e} \neq P_{\bar{\nu}_e \rightarrow \bar{\nu}_e}$ !

With the neutrino mixing in the medium described by  $U'$  and  $\bar{U}'$ , the CP violating I's of Eq. (12) are replaced by

$$I_{\alpha\beta;jk} \Rightarrow \begin{cases} I'_{\alpha\beta;jk} = 4Im[U'_{\alpha j} U'_{\beta j}^* U'_{\alpha k} U'_{\beta k}] \\ \bar{I}'_{\alpha\beta;jk} = 4Im[\bar{U}'_{\alpha j} \bar{U}'_{\beta j}^* \bar{U}'_{\alpha k} \bar{U}'_{\beta k}] \end{cases} \quad (24)$$

It is possible to prove [6] that a real diagonal matter term, as dictated by the Standard Model in Eq. (19), implies the necessary and sufficient condition

$$I_{\alpha\beta;jk} = 0 \iff I'_{\alpha\beta;jk} = \bar{I}'_{\alpha\beta;jk} = 0 \quad (25)$$

This means that the identification of non-vanishing  $I', \bar{I}'$  in matter is still a true signal of CP-violation in Nature. The CP-odd asymmetries contain, however, additional terms which are an undesired background

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta} &= \sum_k |U'_{\beta k}|^2 |U'_{\alpha k}|^2 + 2 \sum_{k>j} Re[U'_{\alpha j} U'_{\beta j}^* U'_{\alpha k} U'_{\beta k}] \cos \frac{\Delta \tilde{m}_{\nu kj}^2}{2E} L \\ &\quad + \frac{1}{2} \sum_{k>j} I'_{\alpha\beta;jk} \sin \frac{\Delta \tilde{m}_{\nu kj}^2}{2E} L \\ P_{\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta} &= \sum_k |\bar{U}'_{\beta k}|^2 |\bar{U}'_{\alpha k}|^2 + 2 \sum_{k>j} Re[\bar{U}'_{\alpha j} \bar{U}'_{\beta j}^* \bar{U}'_{\alpha k} \bar{U}'_{\beta k}] \cos \frac{\Delta \tilde{m}_{\bar{\nu} kj}^2}{2E} L \\ &\quad - \frac{1}{2} \sum_{k>j} \bar{I}'_{\alpha\beta;jk} \sin \frac{\Delta \tilde{m}_{\bar{\nu} kj}^2}{2E} L \end{aligned} \quad (26)$$

From Eq. (26) it is clear that, in matter, the transition probabilities of neutrinos and antineutrinos are different even if CP is conserved, i.e., for  $I' = \bar{I}' = 0$  one has  $D_{\alpha\beta} \neq 0$ . One would need the explicit separation of the odd functions  $\sin \frac{\Delta \tilde{m}_{kj}^2}{2E} L$  in the oscillation probabilities to measure true CP violation effects.

## 6 T-odd asymmetries

Since the matter contribution to the effective neutrino and antineutrino Hamiltonians is real and the matter density is symmetric along the path of the neutrino beam in terrestrial LBL-experiments, matter effects are T-symmetric. A non-vanishing value of  $T_{\alpha\beta}$  or  $\bar{T}_{\alpha\beta}$ , Eq. (10), in matter can only be due to a fundamental violation of T-invariance. It is straightforward to obtain these T-odd asymmetries

$$T_{\alpha\beta} = \sum_{k>j} I'_{\alpha\beta;jk} \sin \frac{\Delta \tilde{m}_{\nu kj}^2}{2E} L \quad ; \quad \bar{T}_{\alpha\beta} = \sum_{k>j} \bar{I}'_{\alpha\beta,jk} \sin \frac{\Delta \tilde{m}_{\bar{\nu} kj}^2}{2E} L \quad (27)$$

One needs, however, the joint measurements of  $\nu_\mu \rightarrow \nu_e$  and  $\nu_e \rightarrow \nu_\mu$  which probably has to wait for neutrino factories in muon-storage-ring facilities.

## 7 Majorana Neutrinos

If neutrinos are Majorana particles, the main modification for charged current neutrino interaction is that the mixing matrix gets replaced [11] by

$$U^D \rightarrow U^M = U^D P \quad ; \quad P = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & 1 \end{pmatrix} \quad (28)$$

with  $U^D$  the conventional  $U$  complex mixing matrix with 3 mixing angles and 1 CP-phase. One sees that 2 additional CP-phases come into the game. Some care is needed because CP-violation through these new phases means  $\alpha_k \neq 0, \frac{\pi}{2}$ !

The new phases are, however, not operative as long as we consider Dirac flavour oscillations for neutrinos propagating through the Green function  $\langle 0|T\{\psi(x)\bar{\psi}(0)\}|0 \rangle$ . Their manifestation would need the study of "neutrino-antineutrino" propagation mediated by the Green function  $\langle 0|T\{\psi(x)\psi^T(0)\}|0 \rangle$ . Langacker et al. [12] have shown that this conclusion is valid not only for neutrino oscillations in vacuum but for flavour oscillations in matter too. The new phases associated with Majorana neutrinos are not seen in Flavour Oscillations in Matter.

## 8 Outlook

The responses to the questions posed in the Introduction are summarized now:

- CP,T violation in neutrino oscillation is possible in Appearance Experiments, for mixing of 3 or more non-degenerate neutrinos.

- Non-vanishing CP, T violating observables need the active participation of the 3 (different) masses, mixings and CP-phase. In particular,  $\frac{E}{L}$  should be comparable to the smallest  $\Delta m^2$ .

- In LBL experiments, there are large matter effects, inducing fake CP and CPT violation. The identification of true CP violation would need the explicit separation of odd functions of time.

- Matter Effects are T-symmetric, so that a non-vanishing T-odd asymmetry in matter is still a signal of fundamental T-violation.

- If neutrinos are Majorana particles, there is no change for Flavour Oscillations, either in vacuum or in matter.

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